BUILDING A BASIC CIRCULAR PARTICLE ACCELERATOR VIA MAD-X

Kaya Tatar (Bogaziçi University, Turkey)

PURPOSE

The aim of this project is to develop a simple circular particle accelerator via the help of MAD-X to serve as a simple example testbed in the MAD-X documentation.

- MAD-X:

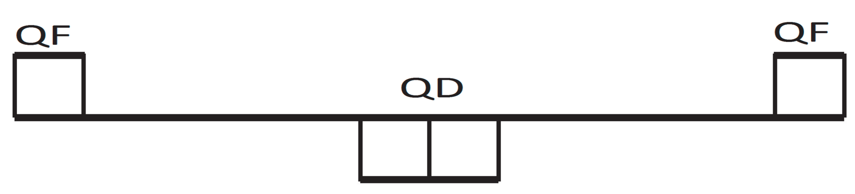
  code used to design and model particle accelerators. It is widely used at CERN and worldwide.

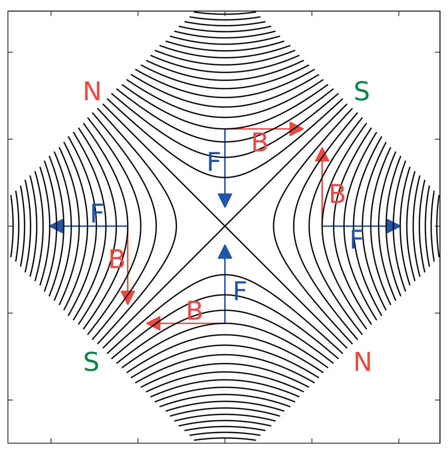
- Basic circular accelerator lattice description

```
qf: quadrupole, l=1./2, k1= +0.6536;
qd: quadrupole, l=1./2, k1= -0.6556;
bd: sbend, l=1.5, angle=2*pi/100;
sf: sextupole, l=0.1, k2= +24.9056;
sd: sextupole, l=0.1, k2= -48.4914;
d1: drift, l=.25;          d2: drift, l=0.15;
cell: line=(qf,sf,d2,bd,d1,qd,qd,sd,d2,bd,d1,qf);
madsync: line=(50*cell);
```

BASIC STRUCTURE

We selected a simple FODO cell, as the basis of the machine.





Magnetic field in quadrupole magnet showing focusing forces

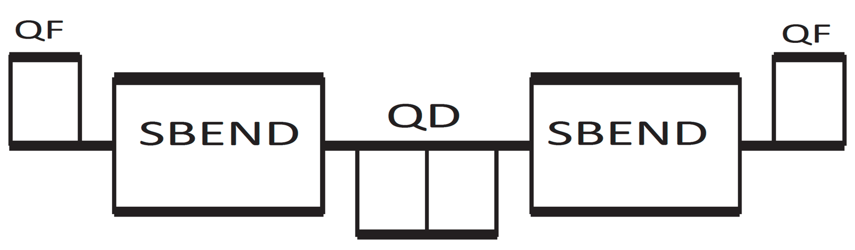
Using "TWISS" we computed μ_x and μ_y.

Phase advance $\mu_x = \int_{cell} \frac{ds}{\beta_x(s)}$

> TWISS : computes optical functions such as β, α, μ, dispersion etc.

Add bending magnets (dipoles) :

- Place into the free space
- Set the total bend angle of the cell to be 2π / 50.

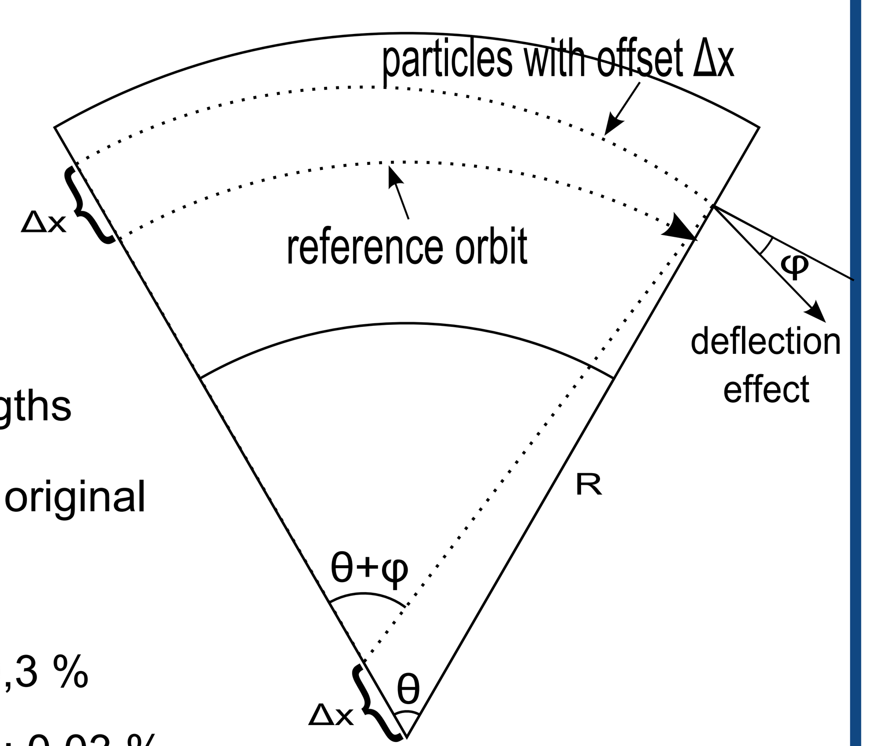


Compute μ_x and μ_y again:

- μ_y does not change
- μ_x changes due to weak focusing

A particle entering the dipole with an horizontal displacement Δx with respect to the reference orbit will exit the dipole with a small deflection angle φ wrt. the reference orbit.

→ weak focusing



Change of φ wrt. Δx : $\frac{\partial\varphi}{\partial\Delta x} \approx \frac{L}{R^2}$,

$L = R\theta$ is the arc length of the dipole.

- Using "MATCH", find quadrupole strengths

Such that μ_x and μ_y are restored to the original value of $1/3$.

- change in focusing magnet strength : 0,3 %
- change in defocusing magnet strength : 0,03 %

> MATCH : finds values of parameters of a sequence that satisfy constraints on a set of optical functions.

We closed the machine by connecting 50 copies of the cell.

→ use "SURVEY" command to check the machine is closed.

- sum of all horizontal bending angles is 2π.
- coordinates of the exit phase of the last element are equal to the coordinates of the entry phase of the first element.

> SURVEY : computes the coordinates of all machine elements.

MOMENTUM DEPENDENCE

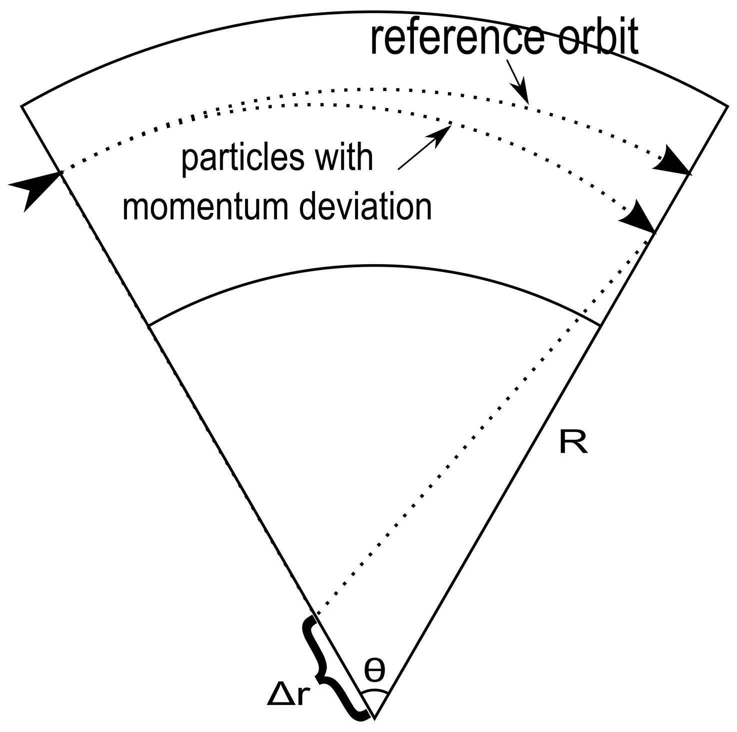
define $\delta p = p - p_0$ where p is the momentum of a specific particle and p_0 is the reference momentum of the beam.

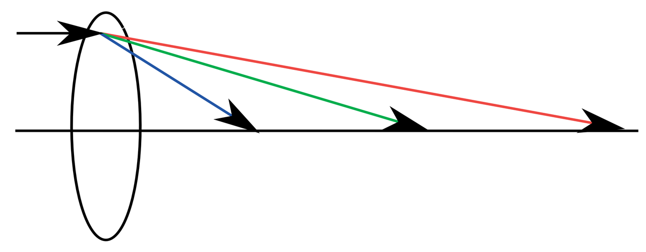
- Inside a ***dipole*** magnet, a particle traverses a circular arc with a radius proportional to its momentum.

Radius of the arc $\rho = \frac{p}{B\,e} = \frac{p_0 + \delta p}{B\,e}$

Over the full machine a particle with $\delta p \neq 0$ has a closed orbit different from that of a particle with reference momentum.

This orbit difference Δx divided by momentum deviation $\delta p / p_0$ is the dispersion function D_x.





reference momentum,
higher momentum,
lower momentum

- Inside a ***quadrupole*** magnet, a particle with $\delta p \neq 0$ is focused differently from a particle with reference momentum.

$K_1 \propto \frac{1}{B\,\rho} \propto \frac{1}{p}$ where $p = p_0 + \delta p$

Integrated over the whole machine, this effect entails that the tune varies with $\delta p/p_0$. The tune is defined as $Q = \frac{1}{2\pi}\int_{turn}\frac{ds}{\beta(s)}$

$Q' = \frac{\partial Q}{\partial(\delta p / p_0)}$ is called *chromaticity.*

CHROMATICITY CORRECTION

Uncorrected chromaticity will lead to beam instabilities. One can compensate the natural chromaticity of the quadrupoles by introducing an equal and opposite focusing strength varying with momentum deviation $\delta p/p_0$.

Solution : The kick provided by a sextupole magnet is proportional to the sextupole strength K_2 times the square of the particle excursion. In the presence of a systematic offset Δx, this gives a kick proportional to $x \cdot \Delta x$. If in turn this offset is proportional to the momentum deviation of the particle ($\Delta x = D_x \cdot \delta p/p_0$), the net effect is chromaticity that can then be made equal and opposite to the chromaticity generated by the quadrupoles.

To correct chromaticity in both planes → need 2 sextupoles

- place the ***focusing*** sextupole where

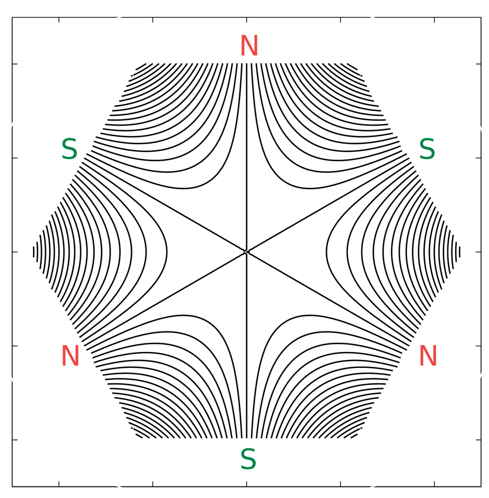
  $\sqrt{\beta_x} \cdot D_x$ is maximum.

  → next to *focusing* quadrupole

- place the ***defocusing*** sextupole where

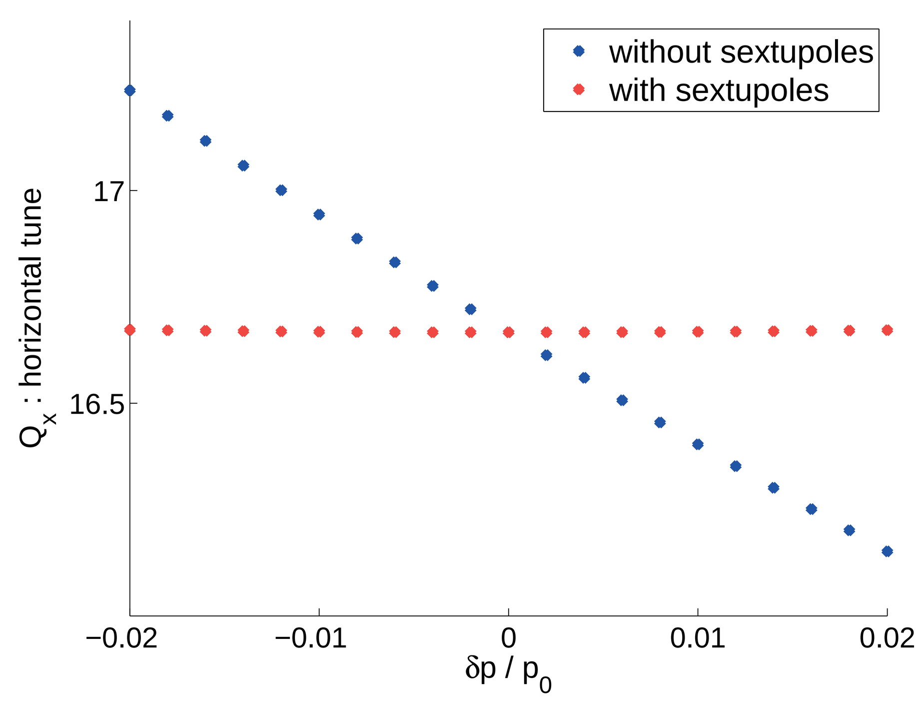
  $\sqrt{\beta_y} \cdot D_x$ is maximum.

  → next to *defocusing* quadrupole



Magnetic field in sextupole magnet

Use "MATCH" with sextupolar strengths as variables to zero total chromaticity in both planes.



In order to check chromaticity, we plotted the horizontal tune as a function of momentum deviation for machines with and without sextupoles.

REFERENCES

[1] Wilson, E. (2001). *Introduction to Particle Accelerators*. Oxford Press.

[2] *MAD - Methodical Accelerator Design*. (n.d.). Retrieved from https://madx.web.cern.ch/madx/

I thank Ghislain Roy and Laurent Deniau for their contributions.